Exercise 32

The cost function for a certain commodity is

$$C(q) = 84 + 0.16q - 0.0006q^2 + 0.000003q^3$$

(a) Find and interpret $C'(100)$.

(b) Compare $C'(100)$ with the cost of producing the 101st item.

Solution

Take the derivative of the cost function to get the marginal cost function.

$$\begin{aligned}\frac{dC}{dq} &= \frac{d}{dq}(84 + 0.16q - 0.0006q^2 + 0.000003q^3) \\ &= 0 + 0.16(1) - 0.0006(2q) + 0.000003(3q^2) \\ &= 0.16 - 0.0012q + 0.000009q^2\end{aligned}$$

Part (a)

Plug in $q = 100$ to get $C'(100)$.

$$C'(100) = 0.16 - 0.0012(100) + 0.000009(100)^2 = 0.16 - 0.12 + 0.09 = 0.13 \ \frac{\$}{\text{item}}$$

This is the rate that the cost is increasing as the 100th item is made; it's also an estimation for the cost of the 101st item.

Part (b)

To obtain the actual cost of the 101st item, subtract $C(100)$ from $C(101)$.

$$\begin{aligned}C(101) - C(100) &= [84 + 0.16(101) - 0.0006(101)^2 + 0.000003(101)^3] \\ &\qquad - [84 + 0.16(100) - 0.0006(100)^2 + 0.000003(100)^3] \\ &= (97.1303) - (97) \\ &= \$0.1303\end{aligned}$$

Use the percent difference formula to see how good the estimation in part (b) is.

$$\frac{0.13 - 0.1303}{0.1303} \times 100\% \approx -0.232535\%$$

Therefore, $C'(100)$ underestimates the actual cost of the 101st item by about 0.23%.